

Short Note on Conditioning Turning Bands Realizations

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Turning bands (TB) method is one of first practical 3-D simulation methods and it is used. TB generates unconditional simulation realizations based on isotropic covariance or variogram models. These realizations can be conditioned by kriging. Although the TB method has several limitations, it is faster than the common sequential Gaussian simulation (SGS) when multiple realizations are required. Programs for TB and conditioning by kriging (CBK) are documented. Examples of TB for conditional simulation are given in the note. Moreover, the CPU times of conditional simulation of TB and SGS are compared.

Introduction

Turning bands method is one of earliest 3-D simulation methods that originated by G. Matheron and developed by A. Journel in early 1970s. Although the sequential simulation method has been popular for many years, turning bands simulation is still used. The turning bands method generates 3-D simulation results from several independent 1-D simulations along lines that can be rotated in 3-D space. This unique way of simulation provides 3-D unconditional realizations. The covariance or variogram from data can be reproduced. Simulation is performed in Gaussian units – the experimental histogram is reproduced by transformation. Conditional simulation of turning bands requires post processing by kriging (Journel and Huijbregts, 1978). This method exactly honors the conditioning data and also preserves the variability of the unconditional simulation realizations. This method has been adapted to conditioning multiple-points structures (Ren, Cunha and Deutsch, 2004). In this note, the focus is on conditional simulation for turning bands method.

The basic principle of the turning band for conditional simulation is illustrated with a 1-D example. A workflow is presented. Programs were created for conditional simulation of TB. 3-D examples are given to demonstrate the conditional simulation and check the programs. Conditional realizations from SGSIM are used to check the conditioning of turning bands.

Turning Bands for Conditional Simulation

In a 3-D space, there are N lines: D_i , $i=1, N$. On each line, there is a 1-D RF $Y(x_{Di})$, $i=1, N$. These N RFs are independent from each other. For each line, we also have a 3-D RF $Z_i(x)$, $i=1, N$. At first, the line D_i is simulated at each point x_{Di} on the line. Usually, use moving average for the 1-D simulation. Second, the simulated value $y(x_{Di})$ at x_{Di} is assigned to all the points inside the slice or band perpendicular to line D_i at x_{Di} :

$$Z_i(x) = y(x_{Di}) \forall x \in R^3$$

Then, at each point x in the 3-D space, sum all the values from the N slices or bands to obtain a realization for this point:

$$Z_s(x) = \frac{1}{\sqrt{N}} \sum_{i=1}^N Z_i(x)$$

After getting the values for all the points in a 3-D model, an unconditional simulation realization is generated.

The conditional simulation of turning band is often done by two kriging runs. First, kriging with the conditioning data to obtain $y_{kc}(x)$. Second, kriging with the unconditional simulated values at these conditioning data locations to obtain $y_{ku}(x)$. Then, calculate the conditional simulation values by the following equation:

$$y_{cs}(x) = y_{uc}(x) + [y_{kc}(x) - y_{ku}(x)]$$

where $y_{uc}(x)$ is the unconditional simulation value from turning bands. The conditional simulation is performed in Gaussian units to ensure histogram reproduction.

The conditional simulation procedure can be easily demonstrated in a 1-D example (Figure 1). At each real data location, the unconditional simulated value is taken out, and the conditioning datum is put in. Near the location, the kriging linear estimators smoothed the change between the real data and the unconditional simulated values outside the range of kriged values. Therefore, after the conditioning, the conditional simulated values at these N data locations will exactly be the real data values. Beyond the range of correction, the conditional simulated values will be the unconditional simulated values.

There are some limitations of turning bands (1) the post processing for conditioning is cumbersome, (2) the 1-D covariances must be worked out separately for each nested structure and covariance shape – they are different in 2-D and 3-D, and (3) only isotropic covariances can be used – anisotropy is introduced by geometric transformation.

Programs for Turning Bands and Conditioning by Kriging

Two programs are available for conditional simulation of turning bands. One is for the turning bands simulation, and another is for the conditioning by kriging. 3-D conditional simulation with multiple realizations can be performed using the two programs.

The `tb3d` program was in the first edition of GSLIB, but not the second edition. This program generates 3-D, unconditional simulations by the turning bands method. The simulated values are standard normal values. Spherical and exponential 3-D variogram models can be used. The dimensioning parameters in `tb3d.inc` should be customized to each simulation study. The parameters required for the program are shown in Figure 2. The number of grids in x and y directions should be equal, $nx = ny$, otherwise, the results may not be realistic.

This conditioning program was created based on the kriging program `kt3d`. It conducts the three successive steps in conditioning by kriging. (1) calculating the residuals or differences between the conditioning data and unconditional simulated values at well locations. (2) perform 3-D kriging using the residuals. The same variogram used in unconditional simulation should be used here. (3) obtain the conditional simulated values by merging the kriged values with the unconditional simulated values. The parameters required for the program are shown in Figure 3. Two types of kriging can be used: simple kriging (SK), input data mean is required and ordinary kriging (OK).

A workflow of the turning bands for conditional simulation is shown on Figure 4.

Examples

A synthetic example on TB for conditional simulation is presented here. The modeled volume is 1000m x 1000m x 250m. Three sets of grids (Table1) were used to demonstrate the TB for conditional simulation and to check the performances.

In this example, 162 regularly spaced well data were available in two layers: layer 1 ($z = 25\text{m}$) and layer 3 ($z = 125\text{m}$). Figure 5 show the conditioning data in each layer and the histogram of the data. Usually, normal score transformation is required for real data.

Figure 6 shows the conditional simulation results of TB using grid set 1. The left column is for layer 1 and right column is for layer 3. The first row is the conditioning data. The secondary row is the unconditional simulation realization from TB. The third row is the conditional simulation realization. As it shown in the Figure, a row of data were picked up at $y=505$ in layer 1 from unconditional realization and conditional realization as well as the conditioning data. These three rows of data were plotted in Figure 7. The dash line is the unconditional results, the red solid line is the conditional results, and the block dots are the conditioning data. We can see that the conditional results are exactly matching the conditioning data, and the shapes of the two lines are quite similarly, which means that the spatial variation of the unconditional simulation realization is well preserved. Therefore, we have the conditional simulation realization for TB.

Performance of TB for conditional simulation

Three sets of grids were used to check the performance of TB for conditional simulation. The conditional simulations with SGSIM were also conducted to compare with the TB. The last row in Figure 6 is SGSIM conditional simulation results for grid set 1. Figures 8 and 9 show the simulation realizations for set 2 and set 3, respectively. The first row is the unconditional realizations from turning band method. The second row is the conditional realizations after conditioning by kriging. The last row is the conditional realizations from SGSIM. Obviously, all the conditional simulation realizations honor the conditioning data. But for larger grids (sets 2 and 3), the TB realizations show some artifacts. Those lines do not show in SGSIM realizations.

Computer cost could be an important factor for conducting simulation with multiple realizations. Therefore, the CPU times of conditional simulations of TB and SGSIM were measured to compare the performances of both methods. The CPU times were compared for generating only one realization. The three sets of grids were used, and the results are listed in Table 2 and plotted in Figure 10. Note for SGSIM, the CPU times change a lot for different search methods. If assign data to nodes, it take only 8 seconds for grids of 50,000. But if not choose this option, use same searching method used in kriging, it will take much longer time than the conditional simulation for TB.

Simulation usually needs multiple realizations. So the CPU times of generating 1, 10, and 100 realizations for grids set 1 were measured. The results are listed in Table 3 and plotted in Figure 11. The SGSIM is using the fastest search method, and we can see that for multiple realizations, the conditional simulation for TB will take much less CPU time than SGSIM when more realizations are needed.

Conclusion

Turning bands method is a fast simulation method when multiple realizations are needed. The conditional simulation is done by conditioning by kriging. Programs for conditional simulation of TB are available.

The aim of this short note was to document the algorithm (and associated software) for teaching purposes. Practitioners may need to be familiar with the method and implement it for testing purposes.

References

- Deutsch, C.V. and Journel, A.G., GSLIB: Geostatistical Software Library and User's Guide, 2nd Edition, Oxford University Press, 1998.
- Journel, A.G. and Huijbregts, C.J., *Mining Geostatistics*, Academic Press, 1978.
- Ren, W., Cunha, L. and Deutsch, C.V., Preservation of Multiple Point Structure when Conditioning by Kriging, CCG report six, 2004.

Grid Defination	Set 1		Set 2		Set 3	
	No. of cells	Cell size	No. of cells	Cell size	No. of cells	Cell size
X	100	10	400	2.5	1000	1
Y	100	10	400	2.5	1000	1
Z	5	50	5	50	5	50
Total grids	50,000		800,000		5,000,000	

Table 1: The three sets of grids used in the example

<i>Max data used</i>		32	-	1, 24	1, 32	ADTN
Grid Size	TB	CBK	TB+CBK	SGSIM	SGSIM	SGSIM
50,000	0.5	36.39	36.89	67.93	93.8	8
800,000	7.42	560.83	568.25	1098.34	1491.68	27.4
5,000,000	46.48	3470.36	3516.84	6845.61	9331.89	170.84

Table 2: CPU times of Conditional simulation of TB and SGSIM with different search methods.

Realizations	Grid Size	TB	CBK	TB+CBK	SGSIM
1	50,000	0.52	36.39	36.91	7.46
10	50,000	4.76	39.72	44.48	73.73
100	50,000	58.48	74.76	133.24	737.39

Table 3: CPU times of Conditional simulation of TB and SGSIM for different simulation realizations.

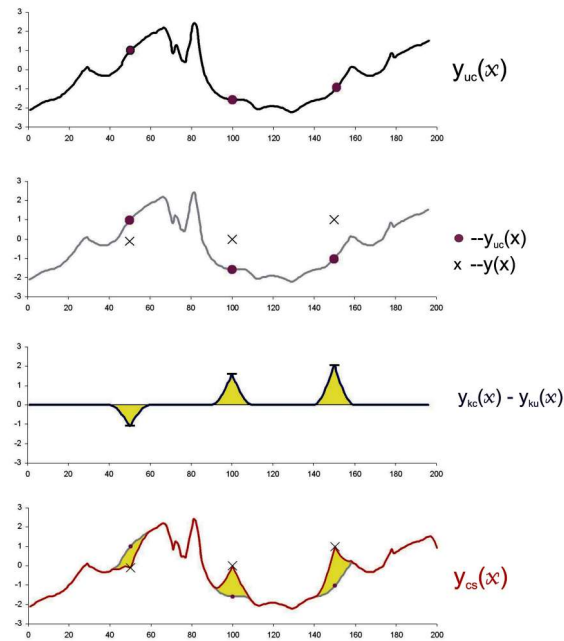


Figure 1: 1-D example of conditioning by kriging.

Parameters for TB3D

```

START OF PARAMETERS:
tb3d.out          \output File for Realization(s)
1                \debugging level: 0,1,2,3
tb3d.dbg          \output File for Debugging
112063           \random number seed
100              \number of simulations
100 5.00 10      \nx,xmn,xsiz
100 5.00 10      \ny,ymn,ysiz
5 25.0 50        \nz,zmn,zsiz
2 0.100          \nst, nugget effect
2 200.0 0.500    \it, aa, cc
0.0 0.0 0.0 1.0 1.0 \ang1,ang2,ang3,anis1,anis2:
1 500.0 0.400    \it, aa, cc
0.0 0.0 0.0 1.0 1.0 \ang1,ang2,ang3,anis1,anis2:

```

Figure 2: An example parameter file for *tb3d*.

Parameters for CBK

```

START OF PARAMETERS:
tb3d.out          - unconditional simulation file
400 1.25 2.5      - nx, xmn, xsiz
400 1.25 2.5      - ny, ymn, ysiz
5 25 50           - nz, zmn, zsiz
100              - number of realizations
conditioning.dat  - file with data
0 1 2 3 4 0       - columns for DH, X, Y, Z, var, sec var
-1.0e21 1.0e21    - trimming limits
0                - debugging level: 0,1,2,3
cbk.dbg          - file for debugging output
cond.out         - file for kriged output
1 1 1            - x, y and z block discretization
1 32             - min, max data for kriging
0               - max per octant (0-> not used)
510. 510. 200.   - maximum search radii
0.0 0.0 0.0      - angles for search ellipsoid
1 2.302          - 0 = SK, 1 = OK
2 0.100          - nst, nugget effect
2 0.500 0. 0. 0. - it,cc,ang1,ang2,ang3
200.0 200.0 200.0 - a_hmax,a_hmin,a_vert
1 0.400 0. 0. 0. - it,cc,ang1,ang2,ang3
500.0 500.0 500.0 - a_hmax,a_hmin,a_vert

```

Figure 3: An example parameter file for *cbk*.

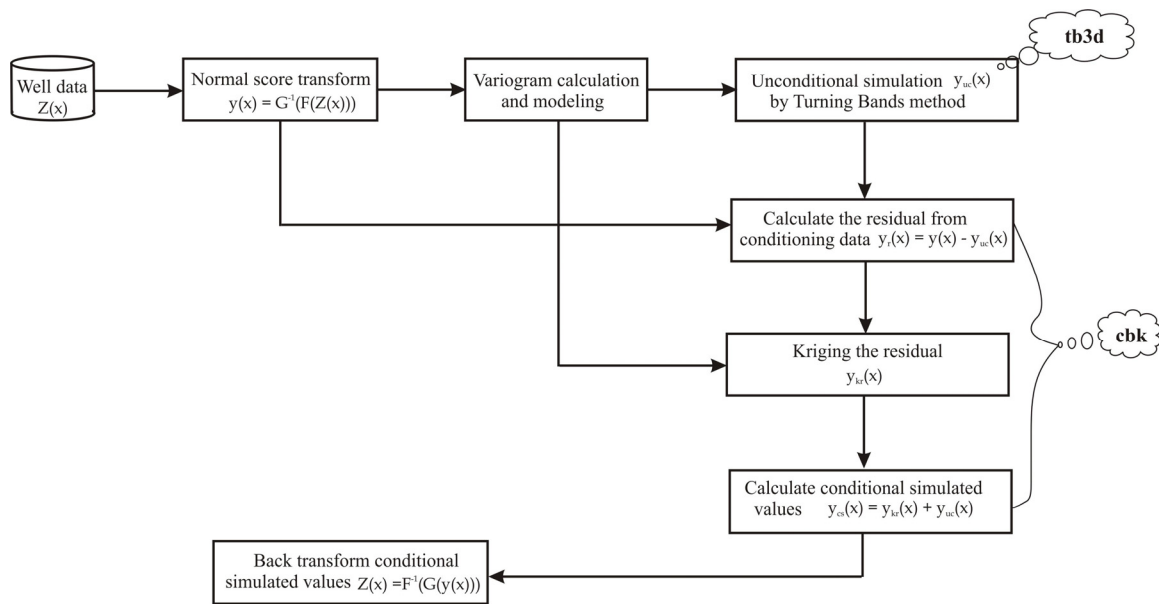


Figure 4: The workflow of turning bands for conditional simulation

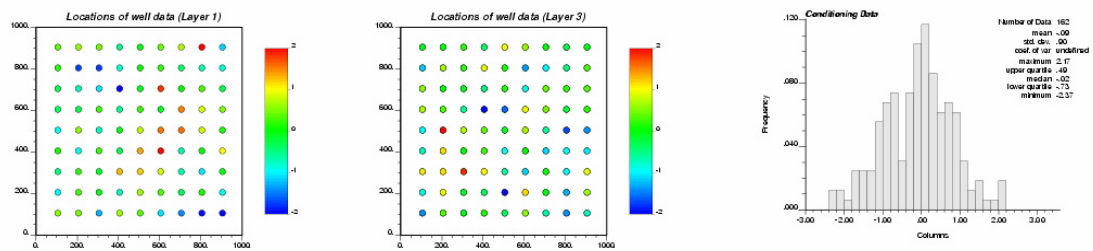


Figure 5: The conditioning data in two layers and the histogram (right) of the data shows a normal distribution.

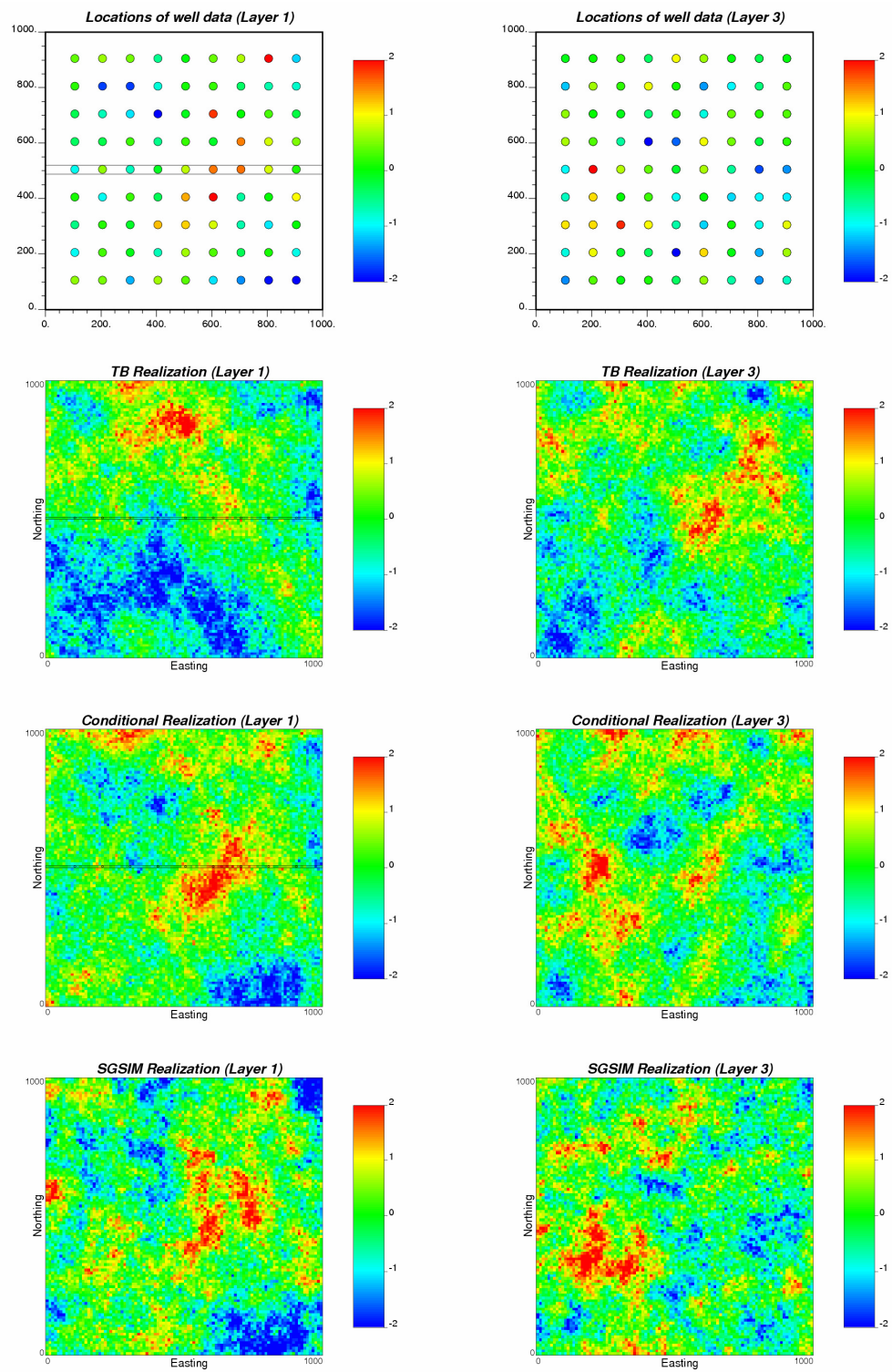


Figure 6: The conditional simulation of turning band with grid setting 1.

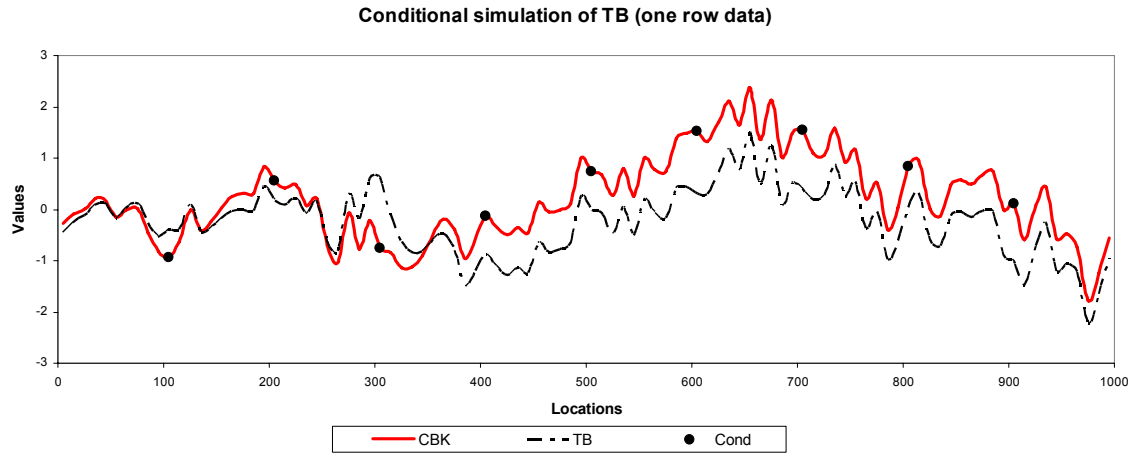


Figure 7: The conditional simulation of turning band with grid setting 1.

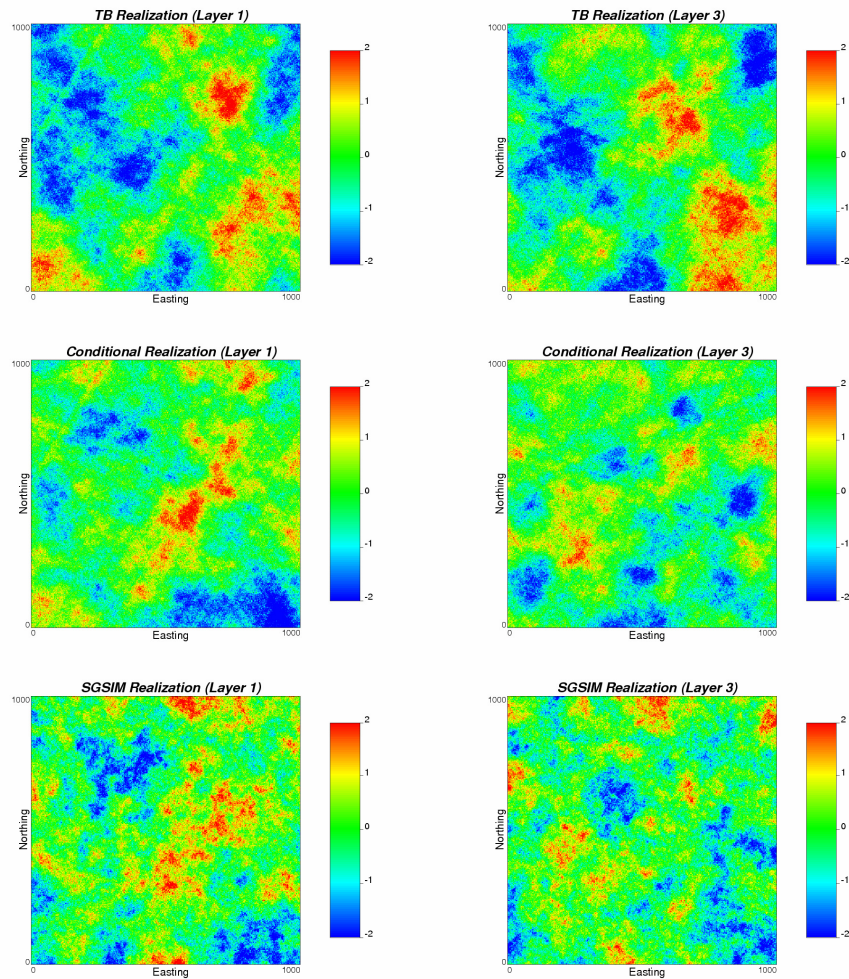


Figure 8: Comparing the conditional realization of turning band with SGSIM at model setting 2.

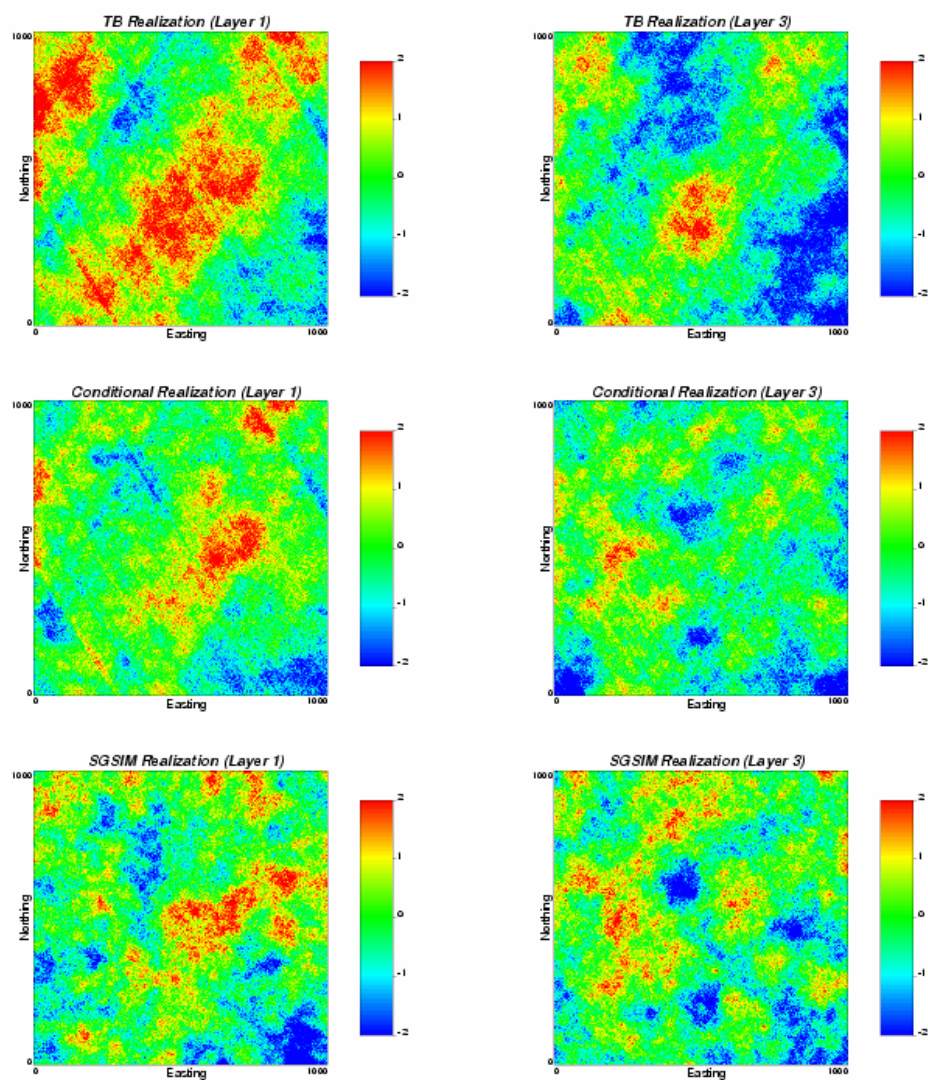


Figure 9: Comparing the conditional realization of turning band with SGSIM at model setting 3.

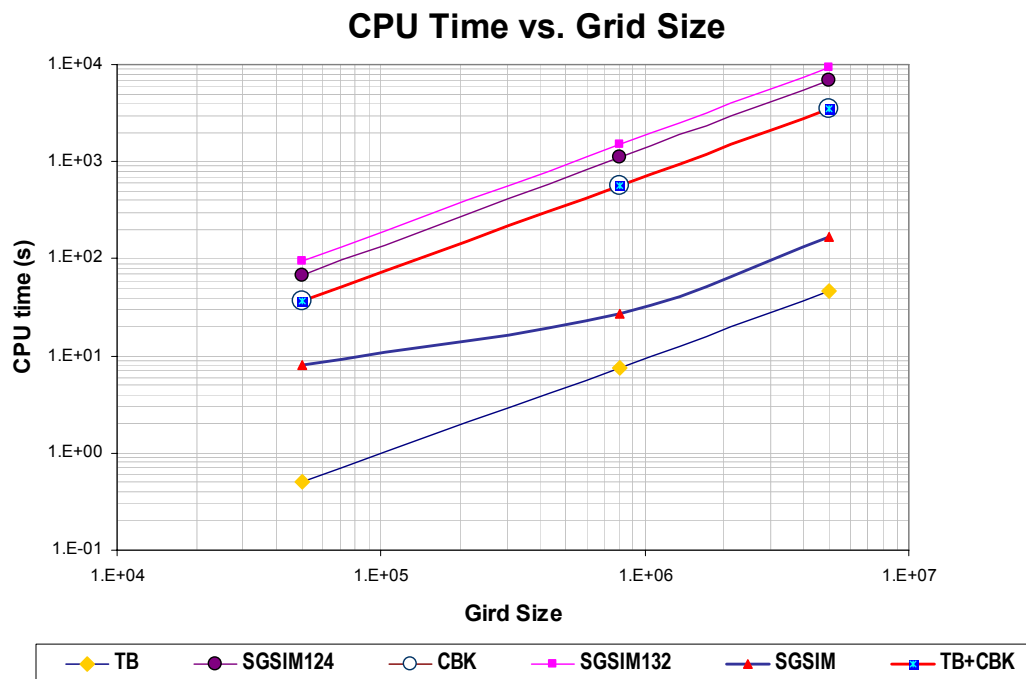


Figure 10: CPU Time comparison.

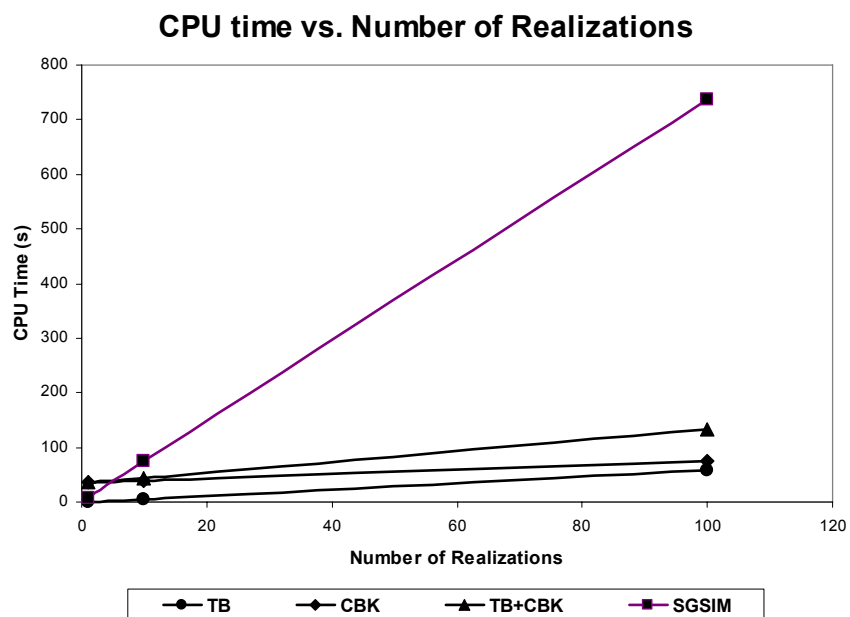


Figure 11: CPU Time Comparison.